## **Cavity-Mediated Electron-Electron Interactions: Renormalizing Dirac States in Graphene**

<u>Hang Liu</u>,<sup>1,\*</sup> Francesco Troisi,<sup>1</sup> Hannes Hübener,<sup>1</sup> Simone Latini,<sup>2,1,\*</sup> and Angel Rubio<sup>1,3,\*</sup>

<sup>1</sup>Max Planck Institute for the Structure and Dynamics of Matter and Center for Free-Electron Laser Science, Luruper Chaussee 149, 22761, Hamburg, Germany; <sup>2</sup>Department of Physics, Technical University of Denmark, 2800 Kgs. Lynby, Denmark; \*Email: <u>hang.liu@mpsd.mpg.de</u>; <u>simola@dtu.dk</u>; <u>angel.rubio@mpsd.mpg.de</u>

Embedding materials in optical cavities has emerged as an alternative strategy for tuning material properties. Accurately modeling the interaction between electrons in materials and photons within dark cavities is crucial for understanding and predicting cavityinduced phenomena. In this work, we develop the photon-free quantum electrodynamical self-consistent Hartree-Fock framework to model the coupling between electrons in crystalline materials and cavity photons. We apply the theoretical approach to investigate graphene coupled to different types of cavity photon modes from vacuum field fluctuation. For a circularly polarized mode, a topological Dirac gap emerges due to cavity-mediated local and nonlocal electron interactions. In contrast, a linearly polarized mode induces a topologically trivial Dirac gap as a result of the cavity-mediated nonlocal electron interactions. Notably, when two symmetric cavity modes are introduced. Dirac cones remain gapless, but Fermi velocity is renormalized through cavity-induced nonlocal electron interactions. Our nonperturbative approach captures the critical role of the cavity-mediated long-range (i.e., nonlocal) electron-electron interactions, which in the case of graphene manifests in the renormalization of Dirac states. The new theoretical framework paves the way for the accurate simulation of light-matter strongly coupled systems beyond perturbation theory, which allows novel cavity-induced phenomena in a broader range of crystalline material systems.